Crucial Knowledge – Stage 1 - Number

BIDMAS

The **order** you do calculations in:

B rackets
I ndices
D ivision
M ultiplication
A ddition
S ubtraction

Place Value

- The 'column values' of numbers

....	Thousands	Hundreds	Tens	Units	Decimal Point	$^1/_{10}$	$^1/_{100}$	$^1/_{1000}$	....

4 Operators

- Addition (or **Sum**) **+**
- Subtraction (or **Difference**) **–**
- Multiplication (or **Product**) **x**
- Division ÷

Negative Numbers

- Adding or subtracting – USE A **NUMBER LINE**
- Multiplying or dividing use the rules

+ x + = +	+ ÷ + = +
+ x - = -	+ ÷ - = -
- x + = -	- ÷ + = -
- x - = +	- ÷ - = +

Fractions Decimals and Percentages

- Different ways of saying part of a whole number
- You can change from one to the other

Prime Numbers

- Have **exactly two factors**
- No other whole numbers, except **1** and **itself** divide into them

Rounding

- Decimal places (column after decimal point)
- Significant Figures (highest value column)

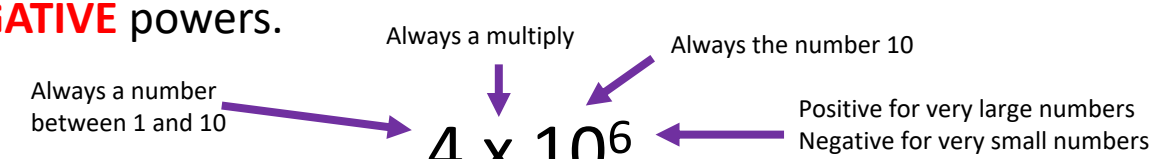
Crucial Knowledge – Stage 1 - Number

Highest Common Factor (HCF)
Lowest Common Multiple (LCM)

- Write down all the factors from the numbers and find the biggest value on both lists – This is the **Highest Common Factor**
- Write down all the multiples of the two numbers and find the smallest on both lists – This is the **Lowest Common Multiple**

Standard Form

- A way of writing very **BIG** or very **SMALL** numbers
- Think **BIG** numbers – distance between planets and **SMALL** numbers – sizes of atoms.
- **BIG** numbers have **POSITIVE** powers and **SMALL** number have **NEGATIVE** powers.



Percentages

- **An amount out of 100**
- **To Calculate a percentage of an amount (What is 40% of £50)**
  Percentage ÷ 100 x amount (40 ÷ 100 x 50 = £20)
- **To change to a percentage (you score 4 out of 5 in a test, what percentage is this?)**
  Amount you got ÷ what it is out of x 100 (4 ÷ 5 x 100 = 80%)

Fractions

- **Multiplying** – Multiply top by top and bottom by bottom.
- **Dividing** - **'Keep Change Flip'.**
- **Addition or Subtraction** – You need same bottom number **(denominator).**

Crucial Knowledge – Stage 2 - Number

Percentage Change

- If a value goes up, it's a percentage increase.
- If a value goes down, it's a percentage decrease.
- We work out percentage of amount and either add it on or subtract it from our starting value
- Or we work out the percentage change by working out the difference in values and dividing by our original value and then multiplying by 100.

Powers

- If we multiply powers we add. $y^3 \times y^4 = y^{(3+4)} = y^7$
- If we divide powers we subtract. $y^{10} \div y^6 = y^{(10-6)} = y^4$
- Anything to the power zero is always 1

Product of Primes

- Any value split into prime numbers MULTIPLIED together.
- First 5 prime numbers are 2, 3, 5, 7 and 11.
- Sometimes we put into a VENN diagram to calculate LCM and HCF.

Inequalities

- Understand inequality symbols $<$ $>$ $\leq$ $\geq$
- List values that satisfy a inequality.
- Show by drawing on a number line values that satisfy inequality.

Estimation

- An answer close to the exact answer.
- All values are rounded to 1 significant figure.
- Follow BIDMAS to get your estimation.

Use of Calculator

- Must be able to use brackets ( ) on calculator to get an answer to multi stage calculations.
- Must be able to use powers on calculator.
- Must be able to use for Standard Form calculations .
- Must be able to use fraction button for all multi tier calculations.
- Must be able to use calculator for percentage calculations.

Crucial Knowledge – Stage 3 - Number

Recurring Decimals

- A decimal with repeating values
- We indicate the repeating numbers with a dot above

$$0.\dot{6} = 0.666666\ldots$$

$$0.\dot{6}5\dot{6} = 0.656656656\ldots$$

$$0.7\dot{1}\dot{6} = 0.7161616\ldots$$

- Must be able to convert recurring decimals to fractions

Fractions

- Mixed number to improper

$$4\frac{1}{3} = \frac{13}{3}$$

$$(4\ x\ 3) + 1 = 13$$

- Improper to mixed number

$$\frac{13}{3} = 4\frac{1}{3}$$

$$13 \div 3 = 4\ remainder\ 1$$

Advanced Powers

- A negative power means reciprocal (”1 over”)
- $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$
- A fractional powers means find a root
- $x^{\frac{1}{2}} = \sqrt{x} \qquad x^{\frac{1}{3}} = \sqrt[3]{x}$
- More complicated fractions require using powers and roots
- $16^{\frac{3}{2}} = \sqrt{16}^{3} = 4^3 = 64$
- Evaluate a negative fractional power in this order

$$16^{-\frac{3}{4}}$$

3rd, 2nd, 1st

- $= (\sqrt[4]{16})^{-3} = 2^{-3} = \frac{1}{8}$

Upper and Lower Bounds

- Upper is slightly above your values
- Lower is slightly below your values
- Using bounds affects calculations – you must find bounds before any calculations
- Example:

Q: A field measures 34m x 28m both measured to the nearest metre. What is the minimum and maximum area the field could have?

A: Bounds

	Upper Bound	Lower Bound
Length (34m)	34.5m	33.5m
Width (28m)	28.5m	27.5m

Maximum Area = 34.5 x 28.5 = 983.25m^2

Minimum Area = 33.5 x 27.5 = 921.25m^2

Crucial Knowledge – Stage 1 – Ratio and Proportion

Ratio as a measure

- A ratio is a comparison of parts
- Use a colon (:) to separate parts of a ratio
- A colon is read as 'to'
- 2 or 3 parts
- Understand the parts add up and stay in proportion

Cancelling ratios

- Like simplifying fractions
- Look for common factors
- Do the same to both parts of the ratio

3:6

÷3 ↓ ↓ ÷3

1:2

Equivalent ratios

- Same values but different numbers
- Values used can get larger, as well as smaller
- Do same to all parts

3:6

x4 ↓ ↓ x4

12:24

Basic unit conversions

- Convert units of length (mm, cm, m, km)
- Be able to convert to common unit before calculating
- Convert units of time
- Convert units of measure (ml, l)
- Convert units of mass (g, kg, t)

Dividing a given ratio

- The question matches the order of items to the order of parts in the ratio. The first thing mentioned gets the first part of the ratio
- Find the total number of parts in the ratio (+)
- Divide the amount to be shared by the total parts (÷)
- Multiply by each part of the ratio (x

Example

Q: Adam and Ben share £45 in the ratio 1:2. Who gets how much?

A: **1 + 2 = 3 parts in total**

£45 ÷ 3 = £15 per part

1:2

x15 ↓ ↓ x15

15:30

Adam gets £15 and Ben gets £30

Crucial Knowledge – Stage 2 – Ratio and Proportion

Unit conversions

- Area conversions
  Use the same conversions as for length, but squared
- Volume conversions
  Use the same conversions as for length, but cubed
- $Speed = \frac{distance}{time}$
- Units for speed include metres per second (m/s) and kilometres per hour (kmph)

Ratio calculations

- Use a ratio to scale measurements up and down
- Examples include using maps and scale drawings
- Size calculations relative to scale and real life

Recipe Scaling

- Work out we have enough to complete
- How much of something do we need

Example:

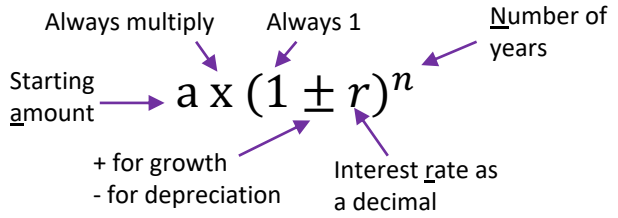
Q: A recipe uses 300g of flour and 150g of butter to make a cake for 4 people. How much of each ingredient is needed to bake a cake for 6 people.

A: 6 ÷ 4 = 1.5 (scale factor).
300g x 1.5 = 450g flour
150 x 1.5 = 225g butter

Crucial Knowledge – Stage 3 – Ratio and Proportion

Interest Calculations

- Compound Interest is an accumulating interest, changing over time, as a growth
- Depreciation is a reduction
- Compound Interest



- A reverse percentage is finding the original value

$$Original = \frac{Final\ Value}{100\ -\%\ Change} x100$$

Proportionality

- Values that have a relationship with each other, as one changes, so does the other one
- Y = kx
- *y* is **directly proportional** to x.

Q: If y=24, then x=8
Work out the value of *y* when x=2.
A: y = kx 24 = kx8 k=3 and so y = 3x
So when x = 2 y = 3 x 2 = 6

Compound Measures

- $Speed = \frac{Distance}{Time}$
- $Density = \frac{Mass}{Volume}$
- $Pressure = \frac{Force}{Area}$

Inverse Proportionality

- Values that have a relationship with each other, as one changes, so does the other, but inverse
- $y = \frac{k}{x}$
- *y* is **inversely proportional** to x.

Q: When y=2, x=3. Work out the value of *y* when x=18
A: $y = \frac{k}{x}$ $2 = \frac{k}{3}$ k = 6 and so $y = \frac{6}{k}$
When x = 18 y = 6 ÷ 18 = 1/3

Crucial Knowledge – Stage 1 – Geometry and Measures

Coordinates

- Remember "along the corridor then up the stairs"
- X and y values written on the axes
- 4 quadrants

Use of Protractor

- Measure angles accurately
- Draw bearings

Terminology Shape

- Edge – Where 2 faces meet
- Vertices – Where 3 faces meet
- Face – side of a 3d shape
- Quadrilateral – a 4 sided polygon
- Polygon – a 2d shape with straight sides
- Acute – an angle less than 90°
- Obtuse – an angle between 90° and 180°
- Reflex – an angle more than 180°

Area and perimeter

- Perimeter is distance around shape
- Area is space inside a shape (2D), measure in square units
- Rectangle $Area = length \times width$
- Triangle $Area = \frac{1}{2}(base \times height)$
  Only use diagonals for perimeter
- Trapezium $Area = \frac{1}{2}(a + b) \times height$
  Only use diagonals for perimeter
- Circle $Area = \pi \times radius^2$
  $Circumference = 2\pi \times radius$

Circumference is the perimeter of a circle

Angle Reasoning

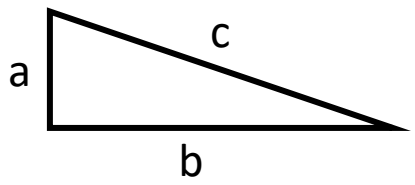
- Angles on straight line = 180°
- Angles in a triangle = 180°
- Vertically opposite angles are always equal
- Angles in quadrilateral = 360°
- Angles at a point = 360°

Types of Triangles

- Scalene – all sides and angles are different
- Isosceles – 2 sides and angles are the same
- Equilateral – 3 sides and angles are the same
- Right – contains a right angle

Crucial Knowledge – Stage 2 – Geometry and Measures

Pythagoras



- $a^2 + b^2 = c^2$
  Square root c^2 to find Hypotenuse
- $c^2 + a^2 = b^2$
  Square root b^2 to find shorter side

Plans and Elevations

- Images from 3 different directions
- Front, side and plan
- Work out size or volume
- Draw 3 images from a 3D drawing
- Draw a 3D image from 3 plans and elevations

Polygons

- A shape with 3 or more straight sides
- Total Interior Angles = (n-2) x 180
- Interior + Exterior = 180°
- Sum of Exterior = 360°

Basic transformations

- Reflections
  Over straight lines (y=, x=) including diagonals (y=x)
- Rotations
  Direction, Distance and Centre
- Translation

$$\begin{pmatrix} right + \ left - \\ up + \ down - \end{pmatrix}$$

- Enlargement
  Scale factor and Centre

Bearings

- 3 digit format
- Measure clockwise from North, 000°
- Be able to draw and add onto a diagram
- Measure reflex angles using a compass
- Calculations using North for parallel lines

Angles with parallel lines

- F – Corresponding
  Always equal
- Z – Alternate
  Always equal
- C – Co-Interior
  Always add to 180°

Crucial Knowledge – Stage 3 – Geometry and Measures

Loci and Constructions

- Perpendicular line bisector
- Angle bisector
- Basic shading of area that satisfy a LOCI

Circle Theories

- Angle facts relating to things in or around a circle
- 8 circle theorems
- Often include Pythagoras' Theorem and Right angled Trigonometry

Similar Shapes

- Divide 2 similar sides to find a linear scale factor
- Area scale factor is the linear scale factor squared
- Volume scale factor is the linear scale factor cubed
- Be prepared to redraw diagrams to help.

Advanced Volumes

- Sphere $Volume = \frac{4}{3}\pi r^3$
- Hemisphere $Volume = \frac{2}{3}\pi r^3$
- Cone $Volume = \frac{1}{3}(Base\ area \times height)$
- Pyramid $Volume = \frac{1}{3}(Base\ area \times height)$
- Frustrum – a cone with a cone cut of the top. Find the volume of the full cone and subtract the volume of the missing cone

Advanced Transformations

- Negative and Fractional enlargements
- Descriptions of single transformations

Right angled trig

- Identify Hypotenuse, Adjacent and Opposite
- Identify Sin, Cos or Tan function
- $Sin\theta = \frac{opp}{hyp}$ $Cos\theta = \frac{adj}{hyp}$ $Tan\theta = \frac{opp}{adj}$
- Normal function for sides
- Inverse function (Sin^{-1}) etc for angles

Crucial Knowledge – Stage 1 – Algebra

Algebra terminology

- 2y means 2 multiplied by the value of 'y'.

  So if y = 5 then 2y = 2 x 5 = **10**

- y^2 the value of 'y' multiplied by itself.

  So if y = 5 then $y^2 = 5 \times 5 = 25$

Substitution

- We get rid of our letters by putting number in to create an answer.
- We are normally given formula and values to put in, but sometimes we have to create the expression and then put values in.
- We need to know about terminology to do this.

You are told $E = \frac{1}{2} mv^2$

Calculate E when m = 10 and v = 2.5

$E = \frac{1}{2} \times 10 \times 2.5 \times 2.5$

$E = 31.25$

Simplifying – Collecting like terms

- We can only bring **'like terms'** together to simplify the expression
- Rewrite to get your **'like terms together'**

Adding and Subtracting

$4a + 3b + 6a - b = 4a + 6a + 3b - b = 10a + 2b$

$3f^2 + 5g^2 + 3f^2 - 7g^2 = 3f^2 + 3f^2 + 5g^2 - 7g^2 = 6f^2 - 2g^2$

Multiplying and Dividing

$4a \times 6a = 24a^2$ (Multiply numbers and add powers)

$30b^5 \div 5b^2 = 6b^3$ (Divide numbers and subtract powers)

Multiplying out single brackets

- Bracket create an order (BIDMAS)
- Brackets are also an invisible multiply

$6(a + 3) = 6 \times a + 6 \times 3 = 6a + 18$

$5(2b - a) = 5 \times 2b + 5 \times -a = 10b - 5a$

$2m(3m - 5) = 2m \times 3m + 2m \times -5 = 6m^2 - 10m$

Crucial Knowledge – Stage 1 – Algebra

Solving equations

- To get a numerical answer for a letter
- We have to do the same to both sides of the equals sign
- If we move things across the equals sign the operator changes to be opposite

Solve 4y + 1 = 17

Move +1 over to become -1

4y = 17 -1 so 4y = 16

Move x4 over to become ÷4 so y = 16÷4

y = 4

Solve 2(3y + 1) = 20

Expand bracket

2 x 3y = 6y and 2 x 1 = 2 so

6y + 2 = 20 Move +2 over to become -2

6y = 20 – 2 so 6y = 18

Move x6 over to become ÷6 so y = 18÷6

y = 3

Factorising

- The process of putting things into brackets
- We can have numerical or algebraic factors
- The 'best' factor goes on the outside of the brackets
- You can check your answer by expanding bracket

Factorise 10a + 5b

'best' factor is 5 so this goes on outside of brackets 5(??????)

2a + b in brackets because when these are multiplied by 5 you get your 10a and 5b

So 5(2a + b) is answer

Factorise $20a^2 + 4a$

'best' factor is 4 number wise and a algebra wise it is a so this goes on outside of brackets 4a(??????)

5a + 1 in brackets because when these are multiplied by 4a you get your $20a^2$ and 4a

So 4a(5a + 1) is answer

Crucial Knowledge – Stage 2 – Algebra

Expanding Double Brackets – FOIL

- Two brackets with nothing between them
- (x+2)(x+5) – This is a double bracket
- 4(x+2) + 5(x+5) – This is 2 single brackets
- When expanding them think First Outer Inner Last
- To start with, you get 4 terms out of double brackets
- You must simplify to 3 or sometimes 2 values

Straight line graphs

- Remember y = ? (this is horizontal line)
- Remember x = ? (this is vertical line)
- You have to substitute values into equations to plot the graph
- y = mx + c where y = y coordinate, m = gradient (how steep graph is), x = x coordinate and c = intercept (where we cut y axis)
- Parallel lines have same gradients
- Gradient is RISE ÷ RUN a positive number we climb and a negative value we ski down

Linear sequences

- A list of numbers that goes up or down by the same amount each time
- Work out Term to Term rule
- Work out your Zero Term
- Form your equation for the nth term
- A value appears if a sequence, the nth term equation is solved with an integer answer.

Solving linear equations – more advanced

- Fractional or non integer – Follow your normal rules, be prepared to give your answer as a fraction, improper fraction or mixed number. It might be positive or negative.
- x on both side – Before you start identify the smallest algebra term and do the opposite of this to both sides of the equation. Then, follow your rules to solve as normal.

Crucial Knowledge – Stage 3 – Algebra

Simultaneous Equations

- When 2 things happen at the same time, sometimes you have to form the equations
- You can sometimes take one equation away form another to solve
- Sometimes you have to cross multiply equations
- Remember **(SSS) S**igns **S**ame **S**ubtract
- Follow your solving linear equation rules

Quadratic Equation

- Be able to apply equation to solve a quadratic

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Algebraic Fractions

- Apply normal rules of fractions
- Apply normal rules of solving algebra
- Sometimes you simplify by factorising
- Your answer may still be an algebraic fraction

Curved Graphs

- Plot x^2 and x^3 graphs using substitution with a table of values
- A $+ x^2$ equation gives a smiley face and $- x^2$ equation a frown
- Use quadratic graphs to obtain equation answers by drawing on your graph

Factorising Quadratics

- Putting into a set of double brackets
- Look for number to be product of factor pairs
- Look for number before '**x**' to be the sum of factor pairs
- To solve make either bracket equal to zero

Quadratic Sequences

- A list of numbers that goes up or down by a different amount each time
- Look for second tier term to term rule each multiple of 2 is one x^2
- Work out first 5 values to this amount of x^2 then solve linear sequence that is the difference between this and original sequence

Crucial Knowledge – Stage 1 – Data and Probability

Mean, median, mode and range

- You must be able to get measures from a list of values or values in a frequency table

- MEAN = Total of values ÷ Number of values
- MEDIAN – The middle value when written in size order
- MODE – The value that occurs the most often
- RANGE – Maximum value – Minimum value

Sample space diagrams

- A list of all possible outcomes from an event. We use this to help calculate probabilities

Probability and relative frequency

- A list of all probabilities adds up to 1
- Relative frequency = $\frac{Times\ occured}{Number\ of\ trials}$

Displaying data

Bar Chart

Pie Chart

Pictogram

Car	🚗🚗🚗🚗🚗
Bus	🚗🚗🚗🚗
Lorry	🚗🚗
Bike	🚗🚗🚗
Van	🚗🚗🚗

Stem and Leaf

Stem	Leaf
1	2 3
2	1 7
3	3 4 5 7 7
4	0 0 1

Interpreting data

- Get values from bar charts
- Get values from pie Charts
- Use key to get values from Stem and Leaf diagram
- Use key to get values from Pictogram

Probability definition and scale

- Outcome – A possible result of an experiment
- Event – A set of outcomes
- Impossible – An outcome that cannot happen
- Certain – An event that must happen

Crucial Knowledge – Stage 2 – Data and Probability

Drawing pie charts

- Angles in a pie chart
  $= \frac{Frequency}{Total\ frequency} x\ 360$
- Use a protractor and ruler to draw accurately

Probability trees

- Used to show outcomes of multiple events
- All branches add up to 1
- Multiply along branches to find probabilities
- Add multiple routes through tree

Stem and leaf diagrams

- Pick correct stems
- Leaves are always single digits
- Ascending order
- Use of key
- Obtain mean, median, mode and range from diagram

Grouped data

- Find Mean from a frequency table
- Find Estimated Mean from grouped frequency table
- Calculated Modal class interval
- Calculate Median class interval
- A class interval means a group of data

Two way tables

- Values add up vertically and horizontally
- Totals can be given but may need to be calculated
- Used to simplify information

Mean, median, mode and range with missing values

- Be able to calculate missing values from a data set when given some of the values.

Example: The mean of the following 5 numbers is 9:

[6] [7] [?] [11] [13]

What is the missing number?

Total value = 5 x 9 = 45

Known total = 6 + 7 + 11 + 13 = 37

Missing value = 45 – 37 = 8

Crucial Knowledge – Stage 3 – Data and Probability

Probability trees with non replacement

- Draw a probability without being asked to
- Change probability on 2nd and potentially 3rd event. Use the information given to determine new probabilities. Make sure all branches add up to 1
- Use tree to calculate complicated event outcomes by multiplying along branches

Listing Number of Outcomes

- Be able to list number of outcomes from a written information:

Example:

Q: A menu contains 3 starters, 5 mains and 2 desserts. How many different 3 course meals can be ordered?

A: 3 x 5 x 2 = 30 different 3 course meals.

Box and Whisker Plots

- Displays 5 key pieces of information.
  1. Minimum
  2. Lower Quartile (Q_1)
  3. Median (Q_2)
  4. Upper Quartile (Q_3)
  5. Maximum
- IQR = $Q_3 - Q_1$
- Draw a box plot
- Compare box plots by stating which median is larger and which IQR is wider.

Cumulative Frequency curves

- Plot cumulative frequencies against interval's upper value
- Hand drawn curve that passes through all points
- Draw on to obtain values using horizontal and vertical lines
- Understand value you are after is sometimes above or below your drawn on value

Histograms

- Looks like a bar chart with different width bars

$$frequency\ density = \frac{frequency}{class\ width}$$

- Complete a table of values
- Draw or complete a histogram
- Find an estimated mean or median by reading values from a histogram